THE OHIO STATE UNIVERSITY
FISHER COLLEGE OF BUSINESS

Illiquid Assets and Smoothed Returns

Andrei S. Gonçalves
Ohio State

December 2024

THE WALL STREET JOURNAL.

English Edition ▾ | Print Edition | Video | Audio | Latest Headlines | More ▾

Latest World Business U.S. Politics Economy Tech **Markets & Finance** Opinion Arts Lifestyle Real Estate Personal Finance Health

WEALTH MANAGEMENT

Considering Alternatives Investments? Factor in Illiquidity

Questions you need to ask before jumping into hedge funds, private real-estate or other deals

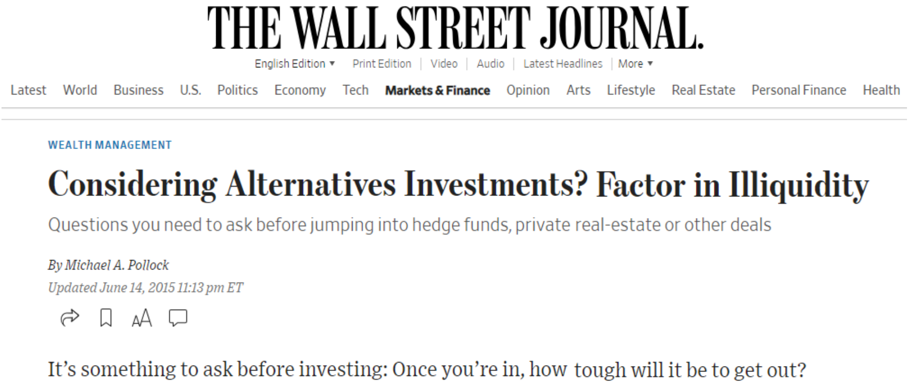
By Michael A. Pollock

Updated June 14, 2015 11:13 pm ET

It's something to ask before investing: Once you're in, how tough will it be to get out?

- Illiquidity induces (at least) two major issues

- This talk will focus on (1)



THE WALL STREET JOURNAL.

English Edition ▾ | Print Edition | Video | Audio | Latest Headlines | More ▾

Latest World Business U.S. Politics Economy Tech **Markets & Finance** Opinion Arts Lifestyle Real Estate Personal Finance Health

WEALTH MANAGEMENT

Considering Alternatives Investments? Factor in Illiquidity

Questions you need to ask before jumping into hedge funds, private real-estate or other deals

By Michael A. Pollock

Updated June 14, 2015 11:13 pm ET

It's something to ask before investing: Once you're in, how tough will it be to get out?

- Illiquidity induces (at least) two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- This talk will focus on (1)

THE WALL STREET JOURNAL.

English Edition ▾ | Print Edition | Video | Audio | Latest Headlines | More ▾

Latest World Business U.S. Politics Economy Tech **Markets & Finance** Opinion Arts Lifestyle Real Estate Personal Finance Health

WEALTH MANAGEMENT

Considering Alternatives Investments? Factor in Illiquidity

Questions you need to ask before jumping into hedge funds, private real-estate or other deals

By Michael A. Pollock

Updated June 14, 2015 11:13 pm ET

It's something to ask before investing: Once you're in, how tough will it be to get out?

- Illiquidity induces (at least) two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- This talk will focus on (1)

THE WALL STREET JOURNAL.

English Edition ▾ | Print Edition | Video | Audio | Latest Headlines | More ▾

Latest World Business U.S. Politics Economy Tech **Markets & Finance** Opinion Arts Lifestyle Real Estate Personal Finance Health

WEALTH MANAGEMENT

Considering Alternatives Investments? Factor in Illiquidity

Questions you need to ask before jumping into hedge funds, private real-estate or other deals

By Michael A. Pollock

Updated June 14, 2015 11:13 pm ET

It's something to ask before investing: Once you're in, how tough will it be to get out?

- Illiquidity induces (at least) two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- This talk will focus on (1)

THE WALL STREET JOURNAL.

English Edition ▾ | Print Edition | Video | Audio | Latest Headlines | More ▾

Latest World Business U.S. Politics Economy Tech **Markets & Finance** Opinion Arts Lifestyle Real Estate Personal Finance Health

WEALTH MANAGEMENT

Considering Alternatives Investments? Factor in Illiquidity

Questions you need to ask before jumping into hedge funds, private real-estate or other deals

By Michael A. Pollock

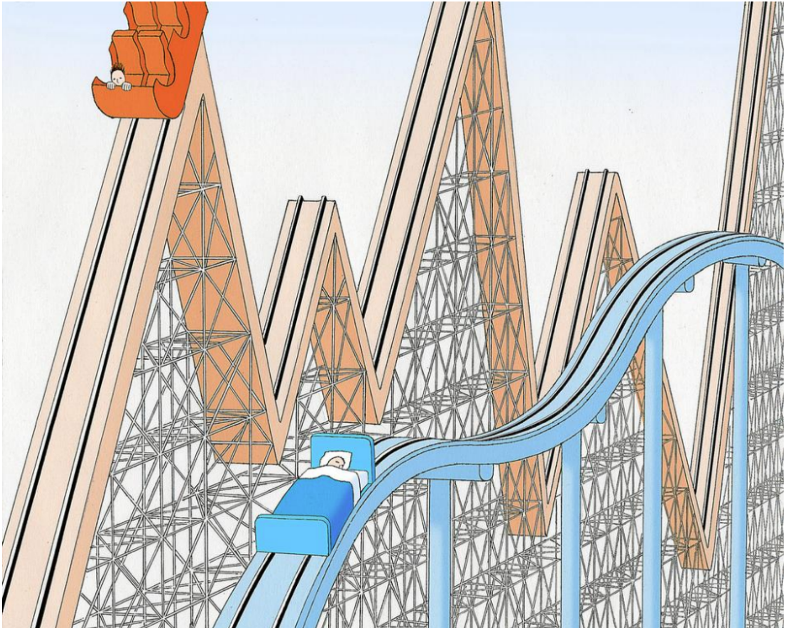
Updated June 14, 2015 11:13 pm ET

It's something to ask before investing: Once you're in, how tough will it be to get out?

- Illiquidity induces (at least) two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- This talk will focus on (1)

Market Value vs Appraised Value

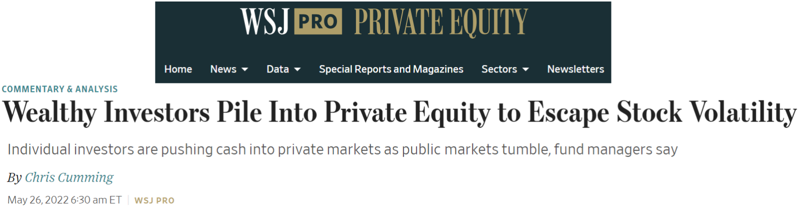
Market Value vs Appraised Value



Low Volatility in Appraised Values

- Key point: low volatility due to (artificially) smoothed returns

Low Volatility in Appraised Values



WSJ PRO PRIVATE EQUITY

Home News Data Special Reports and Magazines Sectors Newsletters

COMMENTARY & ANALYSIS

Wealthy Investors Pile Into Private Equity to Escape Stock Volatility

Individual investors are pushing cash into private markets as public markets tumble, fund managers say

By Chris Cumming

May 26, 2022 6:30 am ET | WSJ PRO

- Key point: low volatility due to (artificially) smoothed returns

Low Volatility in Appraised Values

WSJ PRO PRIVATE EQUITY

Home News Data Special Reports and Magazines Sectors Newsletters

COMMENTARY & ANALYSIS

Wealthy Investors Pile Into Private Equity to Escape Stock Volatility

Individual investors are pushing cash into private markets as public markets tumble, fund managers say

By *Chris Cumming*

May 26, 2022 6:30 am ET | WSJ PRO

INSTITUTIONAL INVESTOR

Subscribe Sign In Register

Portfolio Corner Office Culture Premium Opinion Research Video Innovation

Cliff Asness

Why Does Private Equity Get to Play Make-Believe With Prices?

Investors and managers are playing a dangerous game of "volatility laundering," Cliff Asness writes.

January 6, 2023

- Key point: low volatility due to (artificially) smoothed returns

Low Volatility in Appraised Values

WSJ PRO PRIVATE EQUITY

Home News Data Special Reports and Magazines Sectors Newsletters

COMMENTARY & ANALYSIS

Wealthy Investors Pile Into Private Equity to Escape Stock Volatility

Individual investors are pushing cash into private markets as public markets tumble, fund managers say

By *Chris Cumming*

May 26, 2022 6:30 am ET | WSJ PRO

INSTITUTIONAL INVESTOR

Subscribe Sign In Register

Portfolio Corner Office Culture Premium Opinion Research Video Innovation

Cliff Asness

Why Does Private Equity Get to Play Make-Believe With Prices?

Investors and managers are playing a dangerous game of "volatility laundering," Cliff Asness writes.

January 6, 2023

- Key point: low volatility due to (artificially) smoothed returns

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

(2) How do we detect smoothed returns empirically?

(3) Do illiquid assets display smoothed returns empirically?

(4) What is the main driver of smoothed returns?

(5) How do smoothed returns impact performance measurement?

(6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?

  - * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?

  (3) Do illiquid assets display smoothed returns empirically?

  (4) What is the main driver of smoothed returns?

  (5) How do smoothed returns impact performance measurement?

  (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions
  1. (1) What are smoothed returns?
     - * Returns that have less volatility than if marked-to-market
  2. (2) How do we detect smoothed returns empirically?
  3. (3) Do illiquid assets display smoothed returns empirically?
  4. (4) What is the main driver of smoothed returns?
  5. (5) How do smoothed returns impact performance measurement?
  6. (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?

  * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?

  * Look for excessively positive autocorrelation in returns

  (3) Do illiquid assets display smoothed returns empirically?

  (4) What is the main driver of smoothed returns?

  (5) How do smoothed returns impact performance measurement?

  (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

* Returns that have less volatility than if marked-to-market

(2) How do we detect smoothed returns empirically?

* Look for excessively positive autocorrelation in returns

(3) Do illiquid assets display smoothed returns empirically?

(4) What is the main driver of smoothed returns?

(5) How do smoothed returns impact performance measurement?

(6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?

  * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?

  * Look for excessively positive autocorrelation in returns

  (3) Do illiquid assets display smoothed returns empirically?

  * Yes!

  (4) What is the main driver of smoothed returns?

  (5) How do smoothed returns impact performance measurement?

  (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?
    * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?
    * Look for excessively positive autocorrelation in returns

  (3) Do illiquid assets display smoothed returns empirically?
    * Yes!

  (4) What is the main driver of smoothed returns?

  (5) How do smoothed returns impact performance measurement?

  (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

* Returns that have less volatility than if marked-to-market

(2) How do we detect smoothed returns empirically?

* Look for excessively positive autocorrelation in returns

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

(4) What is the main driver of smoothed returns?

* Illiquidity (as opposed to manipulation)

(5) How do smoothed returns impact performance measurement?

(6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

  * Returns that have less volatility than if marked-to-market

(2) How do we detect smoothed returns empirically?

  * Look for excessively positive autocorrelation in returns

(3) Do illiquid assets display smoothed returns empirically?

  * Yes!

(4) What is the main driver of smoothed returns?

  * Illiquidity (as opposed to manipulation)

(5) How do smoothed returns impact performance measurement?

(6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?

  * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?

  * Look for excessively positive autocorrelation in returns

  (3) Do illiquid assets display smoothed returns empirically?

  * Yes!

  (4) What is the main driver of smoothed returns?

  * Illiquidity (as opposed to manipulation)

  (5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

  (6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

* Returns that have less volatility than if marked-to-market

(2) How do we detect smoothed returns empirically?

* Look for excessively positive autocorrelation in returns

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

(4) What is the main driver of smoothed returns?

* Illiquidity (as opposed to manipulation)

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

(6) How can we solve this issue (at least partially)?

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

(1) What are smoothed returns?

- * Returns that have less volatility than if marked-to-market

(2) How do we detect smoothed returns empirically?

- * Look for excessively positive autocorrelation in returns

(3) Do illiquid assets display smoothed returns empirically?

- * Yes!

(4) What is the main driver of smoothed returns?

- * Illiquidity (as opposed to manipulation)

(5) How do smoothed returns impact performance measurement?

- * Understated risk and overstated risk-adjusted performance

(6) How can we solve this issue (at least partially)?

- * Return unsmoothing

(Artificially) Smoothed Returns

- The notion of smoothed returns raises six important questions

  (1) What are smoothed returns?

  * Returns that have less volatility than if marked-to-market

  (2) How do we detect smoothed returns empirically?

  * Look for excessively positive autocorrelation in returns

  (3) Do illiquid assets display smoothed returns empirically?

  * Yes!

  (4) What is the main driver of smoothed returns?

  * Illiquidity (as opposed to manipulation)

  (5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

  (6) How can we solve this issue (at least partially)?

  * Return unsmoothing

Outline

Outline

Defining Smoothed Returns

(1) What are smoothed returns?

- If that is the case, use market values to calculate returns!
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?

- Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!

- For illiquid assets, we very often do not observe market values

- So, illiquid assets often display smoothed returns

- But, ultimately, they are driven by the lack of market values

- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?
- Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?

- ○ Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - ○ Great solution
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?
   - Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
   - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?

- Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?

- Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?
   - Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
   - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?
  - Returns that have less volatility than if marked-to-market
- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:

Defining Smoothed Returns

(1) What are smoothed returns?
  - Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:
  - Returns based on estimated values = "reported returns" (observable)
  - Returns based on market values = "economic returns" (unobservable)

Defining Smoothed Returns

(1) What are smoothed returns?
  - Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:
  - Returns based on estimated values = "reported returns" (observable)
  - Returns based on market values = "economic returns" (unobservable)

Defining Smoothed Returns

(1) What are smoothed returns?

- Returns that have less volatility than if marked-to-market

- If that is the case, use market values to calculate returns!
  - Great solution...for liquid assets
- For illiquid assets, we very often do not observe market values
- So, illiquid assets often display smoothed returns
- But, ultimately, they are driven by the lack of market values
- Defining return terminology:
  - Returns based on estimated values = "reported returns" (observable)
  - Returns based on market values = "economic returns" (unobservable)

Detecting Smoothed Returns (Logic)

(2) How do we detect smoothed returns empirically?

Detecting Smoothed Returns (Logic)

(2) How do we detect smoothed returns empirically?

* Look for excessively positive autocorrelation in returns

Detecting Smoothed Returns (Logic)

(2) How do we detect smoothed returns empirically?

- Look for excessively positive autocorrelation in returns

Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) \; + \; \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset

Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^{o}_{t+1}) \quad = \quad (1-\theta)\cdot log(V_{t+1}) \, + \, \theta \cdot log(V^{o}_{t})$$

V is the economic value of the asset

V^{o} is the reported value of the asset

Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) \, + \, \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset

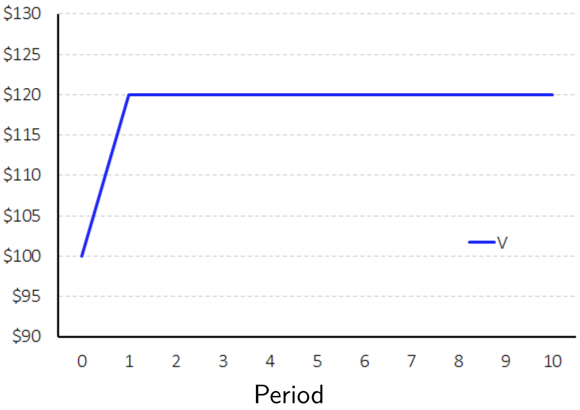
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) = (1-\theta) \cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset



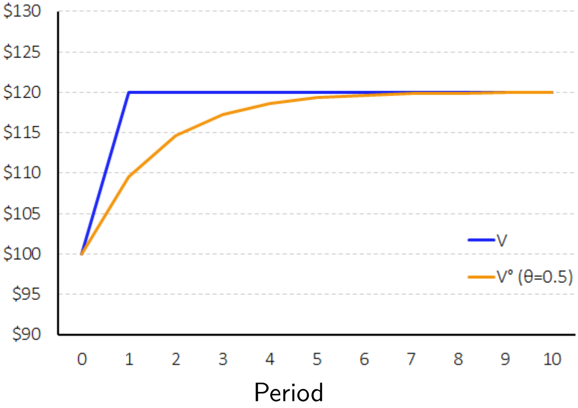
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset



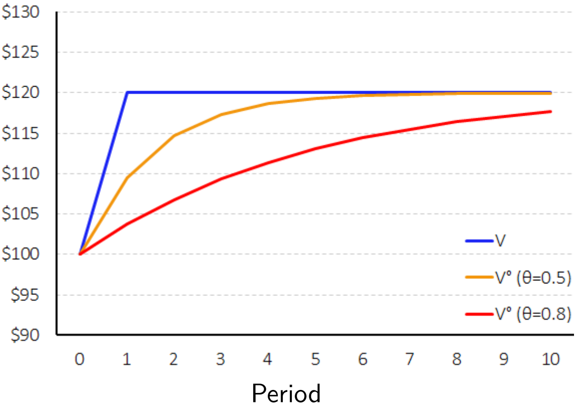
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset



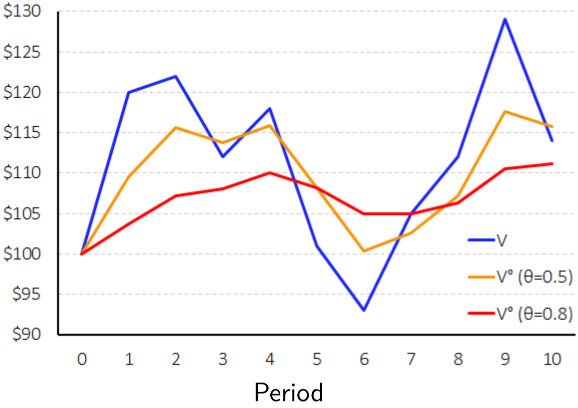
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta)\cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset



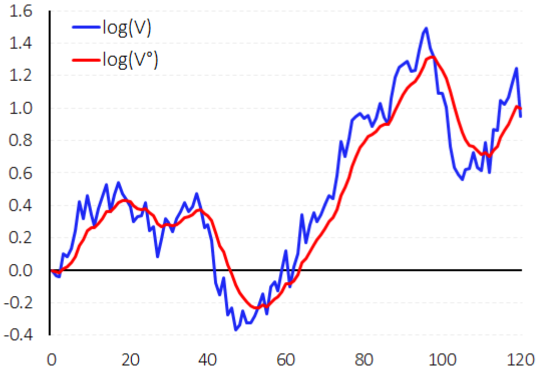
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$log(V^o_{t+1}) \quad = \quad (1-\theta) \cdot log(V_{t+1}) + \theta \cdot log(V^o_t)$$

V is the economic value of the asset

V^o is the reported value of the asset



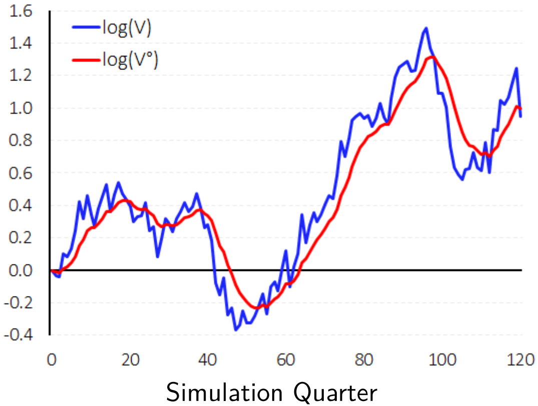
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$r^o_{t+1} \quad = \quad (1-\theta) \cdot r_{t+1} \; + \; \theta \cdot r^o_t$$

r is the economic log return on the asset

r^o is the reported log return on the asset



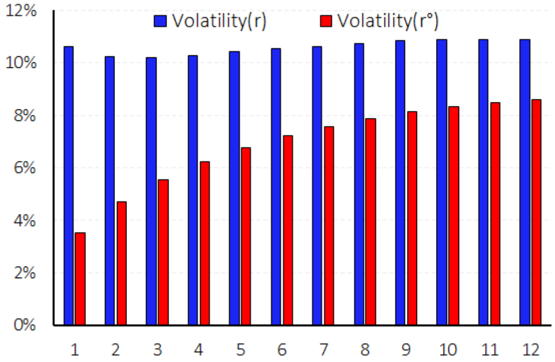
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$r^o_{t+1} \quad = \quad (1-\theta)\cdot r_{t+1} \;+\; \theta \cdot r^o_t$$

r is the economic log return on the asset

r^o is the reported log return on the asset



Returns Frequency (in Quarters)

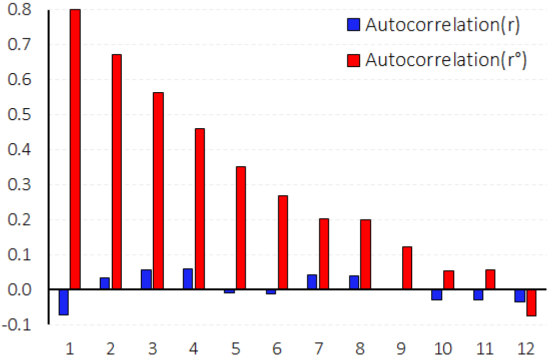
Detecting Smoothed Returns (Logic)

- A simple way to think about smoothed returns is to consider

$$r^o_{t+1} \quad = \quad (1-\theta)\cdot r_{t+1} \;+\; \theta\cdot r^o_t$$

r is the economic log return on the asset

r^o is the reported log return on the asset



Autocorrelation Lag (in Quarters)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

Detecting Smoothed Returns (Evidence)

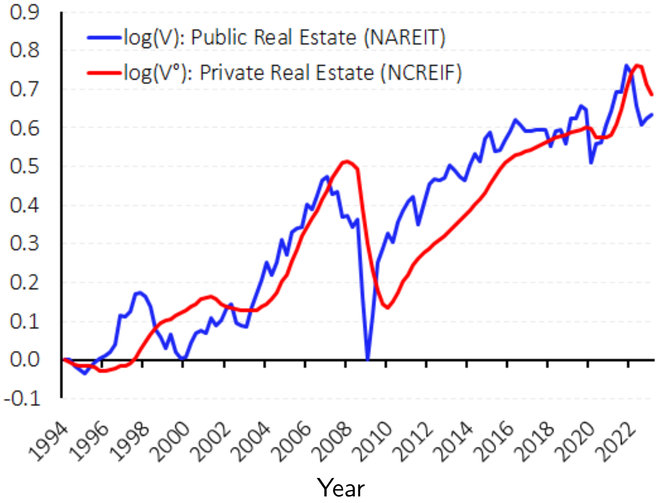
(3) Do illiquid assets display smoothed returns empirically?

- * Yes!

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

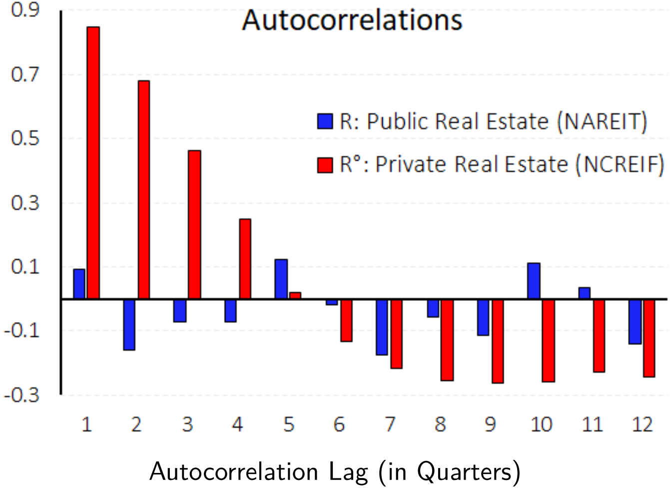
* Yes!



Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

- * Yes!

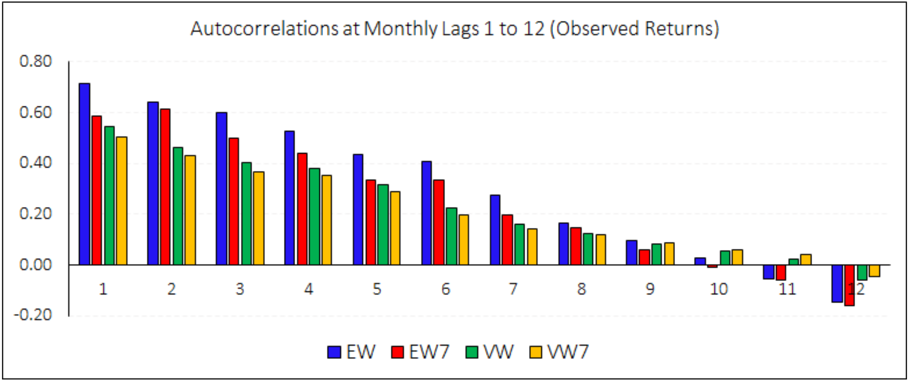


Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

- For NAV REITs:



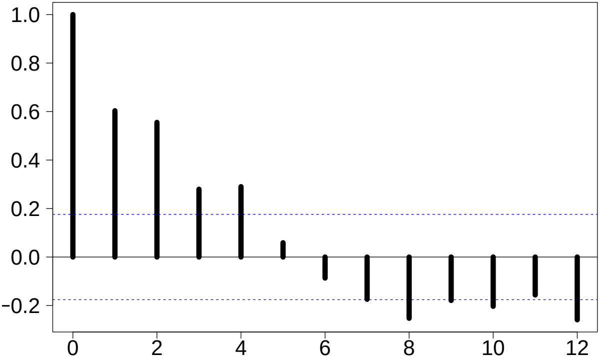
Source: Couts, Gonçalves (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

- For Private Equity Funds (Real Estate):



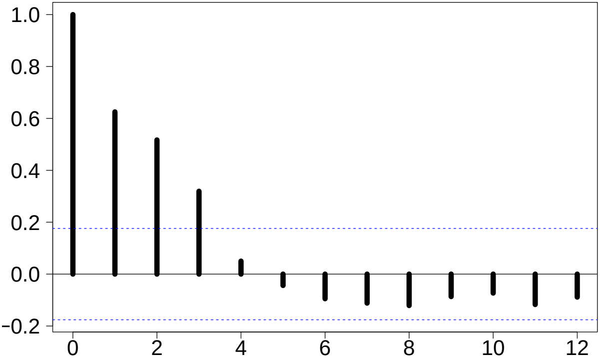
Source: Brown, Gonçalves, Hu (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

- * Yes!

- For Private Equity Funds (Venture Capital):



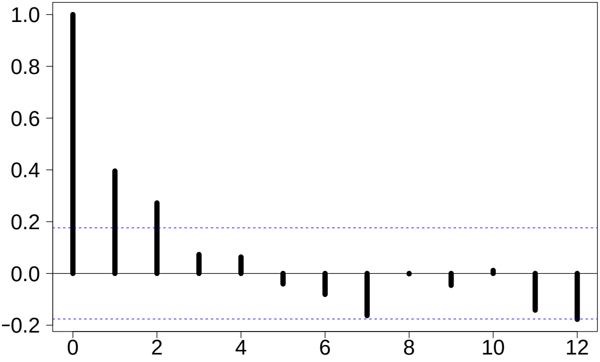
Source: Brown, Gonçalves, Hu (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

- For Private Equity Funds (Buyout):



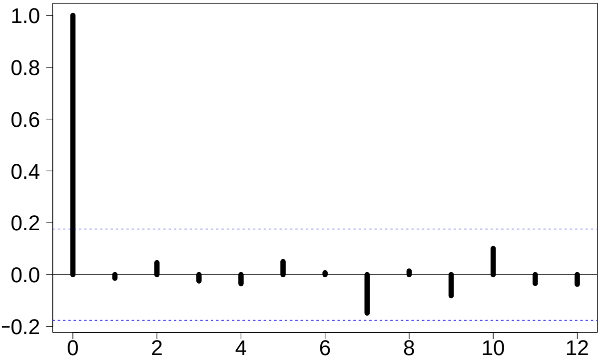
Source: Brown, Gonçalves, Hu (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

- For Public Equities:



Source: Brown, Gonçalves, Hu (2024)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

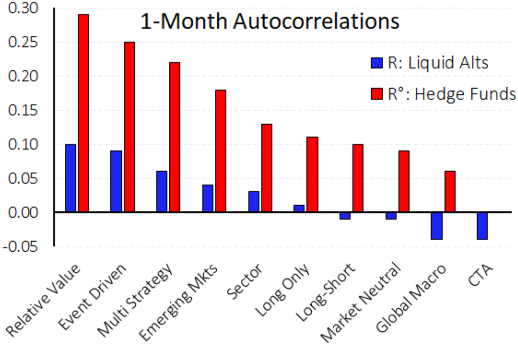
- It is not only about private vs public markets
- For Hedge Funds (assets are publicly traded but often illiquid)

Detecting Smoothed Returns (Evidence)

(3) Do illiquid assets display smoothed returns empirically?

* Yes!

- It is not only about private vs public markets
- For Hedge Funds (assets are publicly traded but often illiquid)



Source: Couts, Gonçalves, Rossi (2020)

Outline

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?
   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to “look good”
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

    * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns
  (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?
   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns
  (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

   * Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns
  (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Main Drivers of Smoothed Returns

(4) What is the main driver of smoothed returns?

* Illiquidity (as opposed to manipulation)

- Managers report AUM based on the value of their asset
- But managers only observe noisy signals of illiquid asset values
- So, optimal estimates of illiquid AUM imply smoothed returns
  (we formalize this statement when discussing return unsmoothing)
- But managers may manipulate reported AUM to "look good"
- Both aspects are present in the data, but illiquidity dominates

Table 2. Lyxor and Main Fund Smoothing

MA(1) model	Main fund	Lyxor	Difference
Average θ_1	0.182	0.121	0.061
t-Statistic	(9.22)	(6.28)	(4.35)

Source: Cao, Farnswoth, Liang, Lo (2017)

Internal vs External Appraisals of Fund Properties

Internal vs External Appraisals of Fund Properties

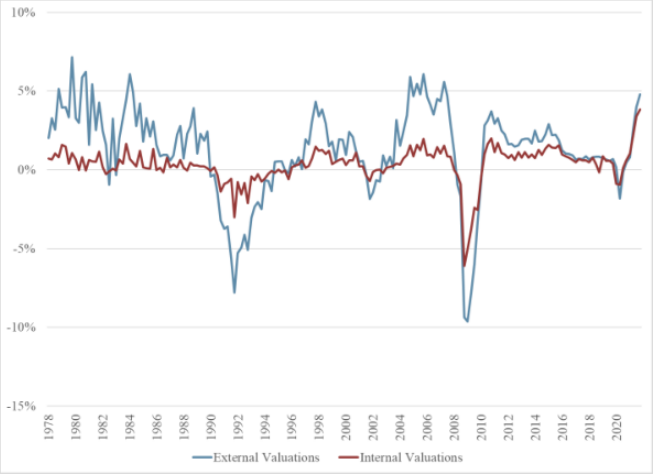


Figure 1
External vs. Internal Indexes (all observations)

Source: Couts (2024)

Internal vs External Appraisals of Fund Properties

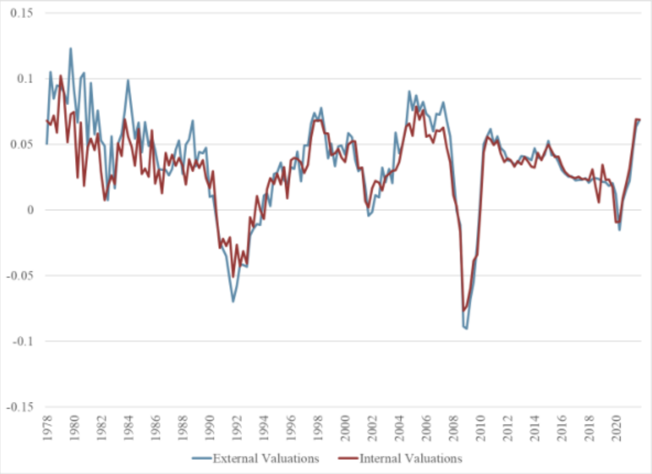
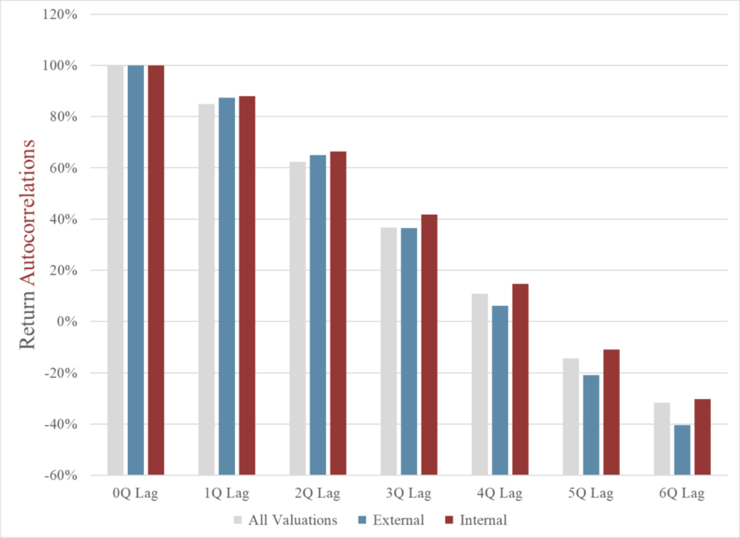


Figure 2
External vs Internal Indexes (no lame valuations)

Source: Couts (2024)

Internal vs External Appraisals of Fund Properties



Source: Couts (2024)

Outline

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

- Consider a simple example:

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta) \cdot R_t + \theta \cdot R_{t-1}$$

$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o \quad = \quad (1-\theta)\cdot R_t \; + \; \theta \cdot R_{t-1}$$

$$R_t \quad = \quad \beta \cdot R_{m,t} \; + \; \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

   * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$
\begin{aligned}
R_t^o &= (1-\theta)\cdot R_t + \theta \cdot R_{t-1} \\
R_t &= \beta \cdot R_{m,t} + \epsilon_t
\end{aligned}
$$

- In this case, the risk exposure estimated using R_t^o is

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\beta$$

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta) \cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \beta$$

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta) \cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \beta$$

- And the estimated alpha is

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\beta$$

- And the estimated alpha is

$$\alpha^o = \mathbb{E}[R^o] - \beta^o \cdot \mathbb{E}[R_m] = (\beta - \beta^o)\cdot\mathbb{E}[R_m] = \theta\cdot\mathbb{E}[R^o]$$

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta) \cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta) \cdot \beta$$

- And the estimated alpha is

$$\alpha^o = \mathbb{E}[R^o] - \beta^o \cdot \mathbb{E}[R_m] = (\beta - \beta^o) \cdot \mathbb{E}[R_m] = \theta \cdot \mathbb{E}[R^o]$$

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\beta$$

- And the estimated alpha is

$$\alpha^o = \mathbb{E}[R^o] - \beta^o \cdot \mathbb{E}[R_m] = (\beta - \beta^o)\cdot\mathbb{E}[R_m] = \theta \cdot \mathbb{E}[R^o]$$

- So, $\beta^o < \beta$ and $\alpha^o > 0$

Effect of Smoothed Returns on Performance Measurement

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- In this case, the risk exposure estimated using R_t^o is

$$\beta^o = \frac{\mathbb{C}ov[R_t^o, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\frac{\mathbb{C}ov[R_t, R_{m,t}]}{\mathbb{V}ar[R_{m,t}]} = (1-\theta)\cdot\beta$$

- And the estimated alpha is

$$\alpha^o = \mathbb{E}[R^o] - \beta^o \cdot \mathbb{E}[R_m] = (\beta - \beta^o)\cdot\mathbb{E}[R_m] = \theta \cdot \mathbb{E}[R^o]$$

- So, $\beta^o < \beta$ and $\alpha^o > 0$

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$
$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- Objection: reported values better reflect "fundamental value"

- Consider $R_{m,t} = -10\%$ with no other movement in markets:

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta\cdot R_{t-1}$$
$$R_t = \beta\cdot R_{m,t} + \epsilon_t$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

  * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$
\begin{aligned}
R_t^o &= (1-\theta)\cdot R_t + \theta \cdot R_{t-1} \\
R_t &= \beta \cdot R_{m,t} + \epsilon_t
\end{aligned}
$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!

- Consider $R_{m,t} = -10\%$ with no other movement in markets:

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

   * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$\begin{aligned} R_t^o &= (1-\theta)\cdot R_t + \theta \cdot R_{t-1} \\ R_t &= \beta \cdot R_{m,t} + \epsilon_t \end{aligned}$$

- Objection: reported values better reflect "fundamental value"
   - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

$$R_t^o = -(1-\theta)\cdot\beta\cdot 10\% \quad \text{and} \quad R_{t+1}^o = -\theta\cdot\beta\cdot 10\%$$

$$\Downarrow$$

$$R_{t+1}^o + R_t^o = -\beta\cdot 10\%$$

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

   * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$R_t^o = (1-\theta)\cdot R_t + \theta \cdot R_{t-1}$$

$$R_t = \beta \cdot R_{m,t} + \epsilon_t$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

$$R_t^o = -(1-\theta)\cdot\beta\cdot 10\% \quad \text{and} \quad R_{t+1}^o = -\theta\cdot\beta\cdot 10\%$$

$$\Downarrow$$

$$R_{t+1}^o + R_t^o = -\beta\cdot 10\%$$

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

 * Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$
\begin{aligned}
R_t^o &= (1-\theta)\cdot R_t + \theta \cdot R_{t-1} \\
R_t &= \beta \cdot R_{m,t} + \epsilon_t
\end{aligned}
$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

$$
R_t^o = -(1-\theta)\cdot\beta\cdot 10\% \quad \text{and} \quad R_{t+1}^o = -\theta\cdot\beta\cdot 10\%
$$

$$
\Downarrow
$$

$$
R_{t+1}^o + R_t^o = -\beta \cdot 10\%
$$

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$
\begin{aligned}
R_t^o &= (1-\theta) \cdot R_t + \theta \cdot R_{t-1} \\
R_t &= \beta \cdot R_{m,t} + \epsilon_t
\end{aligned}
$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

$$
R_t^o = -(1-\theta) \cdot \beta \cdot 10\% \quad \text{and} \quad R_{t+1}^o = -\theta \cdot \beta \cdot 10\%
$$

$$
\Downarrow
$$

$$
R_{t+1}^o + R_t^o = -\beta \cdot 10\%
$$

- If you hold the investment for 2 periods, what matters is β!

The Investor's Risk Exposure is β and not β^o

(5) How do smoothed returns impact performance measurement?

* Understated risk and overstated risk-adjusted performance

- Consider a simple example:

$$\begin{aligned} R_t^o &= (1-\theta)\cdot R_t + \theta \cdot R_{t-1} \\ R_t &= \beta \cdot R_{m,t} + \epsilon_t \end{aligned}$$

- Objection: reported values better reflect "fundamental value"
  - So, the risk that matters is β^o and not β!
- Consider $R_{m,t} = -10\%$ with no other movement in markets:

$$R_t^o = -(1-\theta)\cdot\beta\cdot 10\% \quad \text{and} \quad R_{t+1}^o = -\theta\cdot\beta\cdot 10\%$$

$$\Downarrow$$

$$R_{t+1}^o + R_t^o = -\beta \cdot 10\%$$

- If you hold the investment for 2 periods, what matters is β!

Can Market Time Change the Risk Exposure to β^o?

- Buying and selling illiquid assets incurs large transaction costs (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives

Can Market Time Change the Risk Exposure to β^o?

- Buying and selling illiquid assets incurs large transaction costs
  (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives

Can Market Time Change the Risk Exposure to β^o?

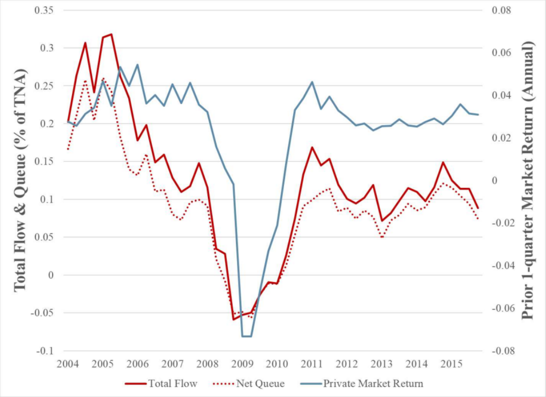
- Buying and selling illiquid assets incurs large transaction costs
  (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives
  - Front-end & back-end loads, lock-in periods, gates, queues...

Can Market Time Change the Risk Exposure to β^o?

- Buying and selling illiquid assets incurs large transaction costs (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives
  - Front-end & back-end loads, lock-in periods, gates, queues...

Can Market Time Change the Risk Exposure to β^o?

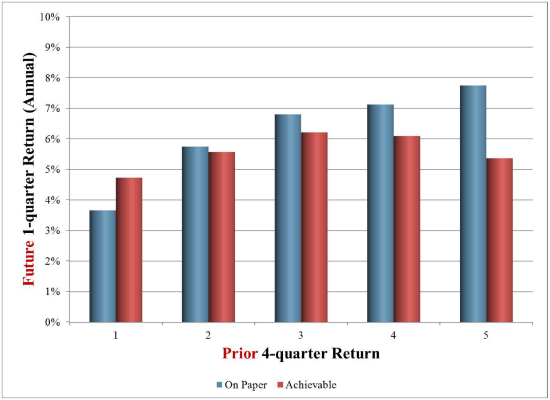
- Buying and selling illiquid assets incurs large transaction costs (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives
  - Front-end & back-end loads, lock-in periods, gates, queues...



Source: Couts (2022)

Can Market Time Change the Risk Exposure to β^o?

- Buying and selling illiquid assets incurs large transaction costs (e.g., Nadauld et al (2019), Boyer et al (2023))
- Funds impose restrictions that limit market timing incentives
    - Front-end & back-end loads, lock-in periods, gates, queues...



Source: Couts (2022)

Outline

Outline

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

- Identification: $R_{j,t} = \mu_j + \eta_{j,t} \quad \text{with} \quad \eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
  (MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + ... + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004)) (MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + \ldots + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + ... + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + \ldots + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} \quad = \quad \mu_j + \theta_j^{(0)} \cdot \eta_{j,t} + \theta_j^{(1)} \cdot \eta_{j,t-1} + \ldots + \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad \theta_j^{(0)} \cdot R_{j,t} \;+\; \theta_j^{(1)} \cdot R_{j,t-1} \;+ \ldots +\; \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} \quad = \quad \mu_j \;+\; \theta_j^{(0)} \cdot \eta_{j,t} \;+\; \theta_j^{(1)} \cdot \eta_{j,t-1} \;+ \ldots +\; \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + ... + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j + \theta_j^{(0)} \cdot \eta_{j,t} + \theta_j^{(1)} \cdot \eta_{j,t-1} + ... + \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + \ldots + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j + \theta_j^{(0)} \cdot \eta_{j,t} + \theta_j^{(1)} \cdot \eta_{j,t-1} + \ldots + \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad \theta_j^{(0)} \cdot R_{j,t} + \theta_j^{(1)} \cdot R_{j,t-1} + ... + \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} \quad = \quad \mu_j + \theta_j^{(0)} \cdot \eta_{j,t} + \theta_j^{(1)} \cdot \eta_{j,t-1} + ... + \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H
- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
  (MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)
- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

$$R_{j,t} \quad = \quad \mu_j + \eta_{j,t}$$

MA(H) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad \theta_j^{(0)} \cdot R_{j,t} \; + \; \theta_j^{(1)} \cdot R_{j,t-1} \; + \ldots + \; \theta_j^{(H)} \cdot R_{j,t-H}$$

where $\sum_{h=0}^{H} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_j^2)$

$$R^o_{j,t} \quad = \quad \mu_j \; + \; \theta_j^{(0)} \cdot \eta_{j,t} \; + \; \theta_j^{(1)} \cdot \eta_{j,t-1} \; + \ldots + \; \theta_j^{(H)} \cdot \eta_{j,t-H}$$

- This is a MA(H): moving average process of order H

- Estimate μ_j and $\theta_j^{(h)}$ by MLE (Getmansky, Lo, Makarov (2004))
(MLE typically imposes $\theta_j^{(0)} = 1$, so divide estimates by $\sum_{h=0}^{H} \theta_j^{(h)}$)

- Get $\eta_{j,t}$ from MA(H) residuals to obtain unsmoothed returns:

$$R_{j,t} \quad = \quad \mu_j \; + \; \eta_{j,t}$$

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} + \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t} \quad \text{with} \quad \eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

- This is an AR(1): autoregressive process of order 1

- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))

- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \; + \; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

- This is an AR(1): autoregressive process of order 1

- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))

- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \; + \; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

- This is an AR(1): autoregressive process of order 1

- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))

- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \; + \; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j + \theta_j \cdot (R^o_{j,t-1} - \mu_j) + \underbrace{(1-\theta_j)\cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1
- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))
- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \ + \ \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j \ + \ \theta_j \cdot (R^o_{j,t-1} - \mu_j) \ + \ \underbrace{(1-\theta_j)\cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1
- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))
- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \ + \ \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j \ + \ \theta_j \cdot (R^o_{j,t-1} - \mu_j) \ + \ \underbrace{(1-\theta_j)\cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1

- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))

- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1 - \theta_j) \cdot R_{j,t} \; + \; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j + \theta_j \cdot (R^o_{j,t-1} - \mu_j) + \underbrace{(1 - \theta_j) \cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1
- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))
- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \;+\; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

$$R^o_{j,t} = \mu_j \;+\; \theta_j \cdot (R^o_{j,t-1} - \mu_j) \;+\; \underbrace{(1-\theta_j)\cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1
- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))
- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

$$R_{j,t} \quad = \quad \mu_j \;+\; \epsilon_{j,t}/(1-\theta_j)$$

AR(1) Unsmoothing

- Reported returns (R^o) reflect past economic returns (R):

$$R^o_{j,t} \quad = \quad (1-\theta_j)\cdot R_{j,t} \; + \; \theta_j \cdot R^o_{j,t-1}$$

where $\sum_{h=0}^{\infty} \theta_j^{(h)} = 1$ (information is eventually incorporated)

- Identification: $R_{j,t} = \mu_j + \eta_{j,t}$ with $\eta_{j,t} \overset{iid}{\sim} \mathcal{D}ist(0, \sigma_j^2)$

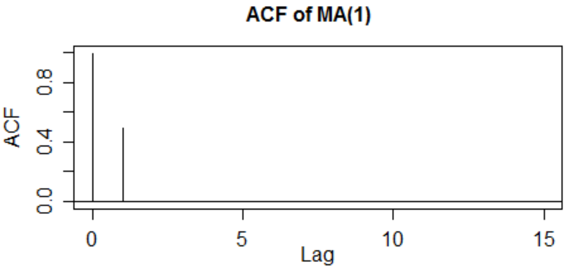
$$R^o_{j,t} = \mu_j \; + \; \theta_j \cdot (R^o_{j,t-1} - \mu_j) \; + \; \underbrace{(1-\theta_j)\cdot \eta_{j,t}}_{\epsilon_{j,t}}$$

- This is an AR(1): autoregressive process of order 1
- We can estimate μ_j and θ_j by OLS (Geltner (1991, 1993))
- Get $\epsilon_{j,t}$ from AR(1) residuals to obtain unsmoothed returns:

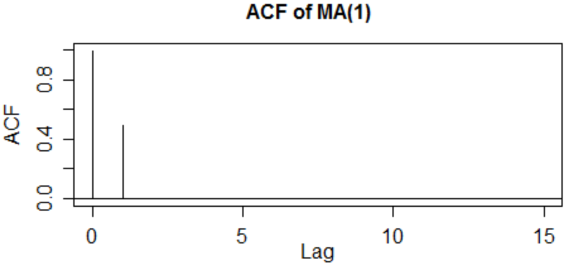
$$R_{j,t} \quad = \quad \mu_j \; + \; \epsilon_{j,t}/(1-\theta_j)$$

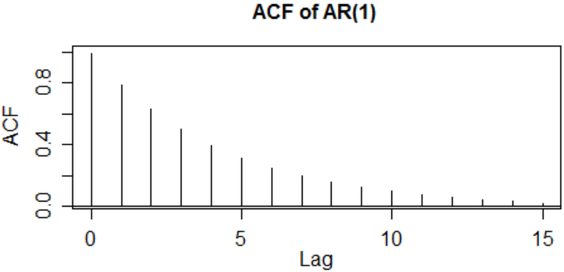
Autocorrelation Functions: MA(1) and AR(1)

Autocorrelation Functions: MA(1) and AR(1)



Autocorrelation Functions: MA(1) and AR(1)

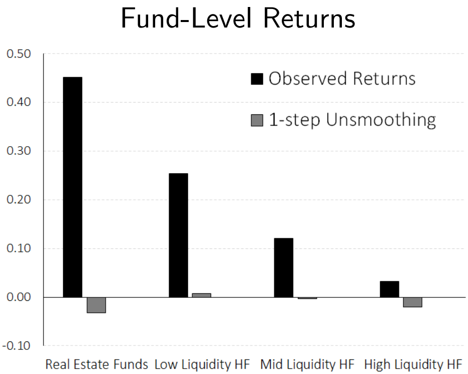




Outline

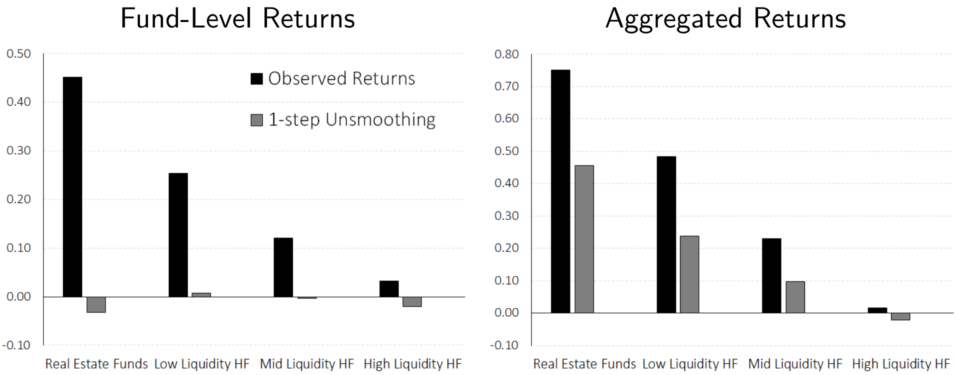
Motivation: Autocorrelations (1 Lag)

Motivation: Autocorrelations (1 Lag)



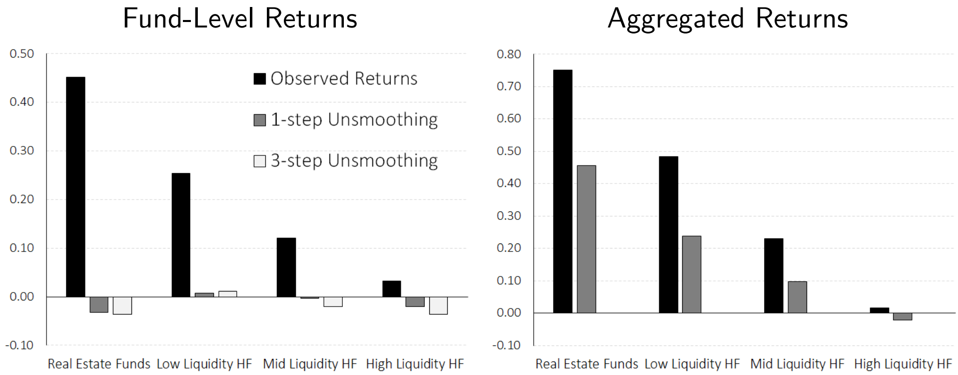
Source: Couts, Gonçalves, Rossi (2020)

Motivation: Autocorrelations (1 Lag)



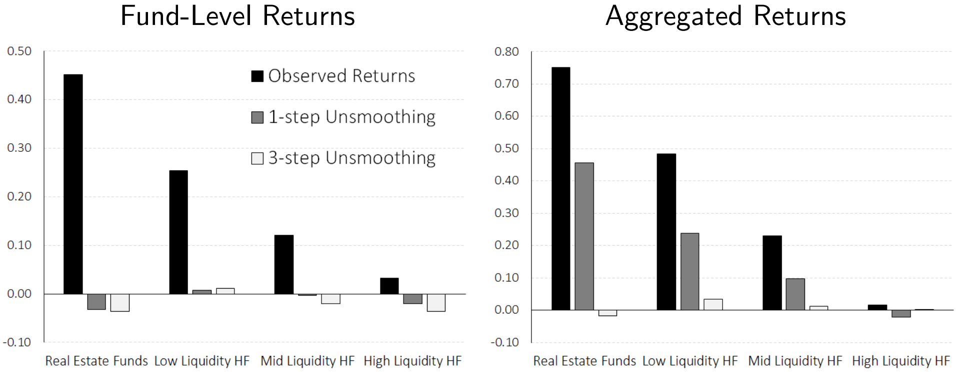
Source: Couts, Gonçalves, Rossi (2020)

Motivation: Autocorrelations (1 Lag)



Source: Couts, Gonçalves, Rossi (2020)

Motivation: Autocorrelations (1 Lag)



Source: Couts, Gonçalves, Rossi (2020)

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h} \\
&= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}
\end{aligned}
$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$\begin{aligned} R^o_{j,t} &= \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \end{aligned}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \pi^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}$s as covariates

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

$$\widetilde{R}^o_{j,t} = \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \psi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h}$$

$$= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

$$\widetilde{R}^o_{j,t} = \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \psi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step MA(H) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$\begin{aligned} R^o_{j,t} &= \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \end{aligned}$$

where $\sum_{h=0}^{H} \phi_j^{(h)} = \sum_{h=0}^{H} \pi_j^{(h)} = 1$

- **Step 1**: $\overline{\eta}_t$ from MA(H) on $\overline{R}^o_t$

$$\overline{R}^o_{j,t} \approx \mu_j + \sum_{h=0}^{H} \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 2**: $\widetilde{\eta}_{j,t}$ from MA(H) on $\widetilde{R}^o_{j,t}$ with $\overline{\eta}s$ as covariates

$$\widetilde{R}^o_{j,t} = \mu_j + \sum_{h=0}^{H} \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} + \sum_{h=0}^{H} \psi_j^{(h)} \cdot \overline{\eta}_{t-h}$$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= (1-\phi_j)\cdot \widetilde{R}_{j,t} + \phi_j \cdot \widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot \overline{R}_t + \pi_j \cdot \overline{R}^o_{j,t-1} \\
&= \mu_j \;+\; \phi_j \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j \cdot (\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
\end{aligned}
$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad \Rightarrow \quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$
- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j\cdot\widetilde{R}^o_{j,t-1} + (1-\pi_j)\cdot\overline{R}_t + \pi_j\cdot\overline{R}^o_{j,t-1}$$

$$= \mu_j + \phi_j\cdot(\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) + \pi_j\cdot(\overline{R}^o_{j,t-1} - \overline{\mu}_j) + \epsilon_{j,t}$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad \Rightarrow \quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$
- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= (1-\phi_j)\cdot \widetilde{R}_{j,t} + \phi_j \cdot \widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot \overline{R}_t + \pi_j \cdot \overline{R}^o_{j,t-1} \\
&= \mu_j \;+\; \phi_j \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j \cdot (\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
\end{aligned}
$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad\Rightarrow\quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$
- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j\cdot\widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot\overline{R}_t + \pi_j\cdot\overline{R}^o_{j,t-1} \\
&= \mu_j \;+\; \phi_j\cdot(\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j\cdot(\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
\end{aligned}
$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad\Rightarrow\quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$
- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$\begin{aligned} R^o_{j,t} &= (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j\cdot\widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot\overline{R}_t + \pi_j\cdot\overline{R}^o_{j,t-1} \\ &= \mu_j \;+\; \phi_j\cdot(\widetilde{R}^o_{j,t-1}-\widetilde{\mu}_j) \;+\; \pi_j\cdot(\overline{R}^o_{j,t-1}-\overline{\mu}_j) \;+\; \epsilon_{j,t} \end{aligned}$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above

- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad\Rightarrow\quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$
\begin{aligned}
R^o_{j,t} &= (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j \cdot \widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot\overline{R}_t + \pi_j \cdot \overline{R}^o_{j,t-1} \\
&= \mu_j \;+\; \phi_j \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \;+\; \pi_j \cdot (\overline{R}^o_{j,t-1} - \overline{\mu}_j) \;+\; \epsilon_{j,t}
\end{aligned}
$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above

- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \qquad \Rightarrow \qquad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$R^o_{j,t} = (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j\cdot\widetilde{R}^o_{j,t-1} + (1-\pi_j)\cdot\overline{R}_t + \pi_j\cdot\overline{R}^o_{j,t-1}$$

$$= \mu_j + \phi_j\cdot(\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) + \pi_j\cdot(\overline{R}^o_{j,t-1} - \overline{\mu}_j) + \epsilon_{j,t}$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above

- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \quad \Rightarrow \quad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$

- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$ where

$$\widetilde{\eta}_{j,t} = \frac{\epsilon_{j,t} - (1-\pi_j)\cdot\overline{\eta}_t}{(1-\phi_j)}$$

3-Step AR(1) Unsmoothing

- R^o reflects past aggregate ($\overline{R}$) and relative ($\widetilde{R}$) returns:

$$\begin{aligned} R^o_{j,t} &= (1-\phi_j)\cdot\widetilde{R}_{j,t} + \phi_j\cdot\widetilde{R}^o_{j,t-1} \;+\; (1-\pi_j)\cdot\overline{R}_t + \pi_j\cdot\overline{R}^o_{j,t-1} \\ &= \mu_j \;+\; \phi_j\cdot(\widetilde{R}^o_{j,t-1}-\widetilde{\mu}_j) \;+\; \pi_j\cdot(\overline{R}^o_{j,t-1}-\overline{\mu}_j) \;+\; \epsilon_{j,t} \end{aligned}$$

where $\epsilon_{j,t} = (1-\phi_j)\cdot\widetilde{\eta}_{j,t} + (1-\pi_j)\cdot\overline{\eta}_t$

- **Step 1**: $\epsilon_{j,t}$ are residuals in the time-series regression above
- **Step 2**: $\overline{\epsilon}_t \approx (1-\overline{\pi})\cdot\overline{\eta}_t \qquad \Rightarrow \qquad \overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$
- **Step 3**: $R_{j,t} = \mu_j + \widetilde{\eta}_{j,t} + \overline{\eta}_t$ where

$$\widetilde{\eta}_{j,t} = \frac{\epsilon_{j,t} - (1-\pi_j)\cdot\overline{\eta}_t}{(1-\phi_j)}$$

Outline

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:
- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal
- The manager reports the Bayesian estimate for the fund value,
- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

- The manager reports the Bayesian estimate for the fund value,

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

- The manager reports the Bayesian estimate for the fund value,

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} \;=\; \mu_j \;+\; v_{j,t-1} \;+\; \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} \;=\; \eta_{j,t} \;+\; u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v^o_{j,t} = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t}|\widehat{\eta}_{j,t}]}_{\theta_j^{(0)}\cdot\widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^{o} = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} \; = \; \mu_j \; + \; v_{j,t-1} \; + \; \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} \; = \; \eta_{j,t} \; + \; u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^o \quad = \quad \mu_j \; + \; v_{j,t-1} \; + \; \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} \;=\; \mu_j \;+\; v_{j,t-1} \;+\; \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} \;=\; \eta_{j,t} \;+\; u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^{o} \;=\; \mu_j \;+\; v_{j,t-1} \;+\; \underbrace{\mathbb{E}[\eta_{j,t}|\widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

$$r_{j,t}^{o} \;=\; v_{j,t}^{o} \;-\; v_{j,t-1}^{o} \;=\; (v_{j,t-1} - v_{j,t-2}) \;+\; \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1})$$

$$=\; \theta_j^{(0)} \cdot r_{j,t} \;+\; (1 - \theta_j^{(0)}) \cdot r_{j,t-1} \;+\; \xi_{j,t}$$

where $\xi_{j,t} = \theta_j^{(0)} \cdot (u_{j,t} - u_{j,t-1})$

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^o = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

$$r_{j,t}^o = v_{j,t}^o - v_{j,t-1}^o = (v_{j,t-1} - v_{j,t-2}) + \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1})$$

$$= \theta_j^{(0)} \cdot r_{j,t} + (1 - \theta_j^{(0)}) \cdot r_{j,t-1} + \xi_{j,t}$$

where $\xi_{j,t} = \theta_j^{(0)} \cdot (u_{j,t} - u_{j,t-1})$

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \qquad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \qquad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v^o_{j,t} = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

$$r^o_{j,t} = v^o_{j,t} - v^o_{j,t-1} = (v_{j,t-1} - v_{j,t-2}) + \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1})$$

$$= \theta_j^{(0)} \cdot r_{j,t} + (1 - \theta_j^{(0)}) \cdot r_{j,t-1} + \xi_{j,t}$$

where $\xi_{j,t} = \theta_j^{(0)} \cdot (u_{j,t} - u_{j,t-1})$

Smoothed Returns with Bayesian Fund Manager: MA(1)

- Consider a simple framework for $v_{j,t} = log(V_{j,t})$:

$$v_{j,t} = \mu_j + v_{j,t-1} + \eta_{j,t} \quad \text{with} \quad \eta_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$$

- At t, the manager learns $v_{j,t-1}$ and observes the noisy signal

$$\widehat{\eta}_{j,t} = \eta_{j,t} + u_{j,t} \quad \text{with} \quad u_{j,t} \sim \mathcal{N}(0, \widehat{\sigma}_j^2)$$

- The manager reports the Bayesian estimate for the fund value,

$$v_{j,t}^o = \mu_j + v_{j,t-1} + \underbrace{\mathbb{E}[\eta_{j,t} | \widehat{\eta}_{j,t}]}_{\theta_j^{(0)} \cdot \widehat{\eta}_{j,t}}$$

where $\theta_j^{(0)} = (1/\widehat{\sigma}_j^2)/(1/\sigma_j^2 + 1/\widehat{\sigma}_j^2)$

- In this case, the reported return is

$$r_{j,t}^o = v_{j,t}^o - v_{j,t-1}^o = (v_{j,t-1} - v_{j,t-2}) + \theta_j^{(0)} \cdot (\widehat{\eta}_{j,t} - \widehat{\eta}_{j,t-1})$$

$$= \theta_j^{(0)} \cdot r_{j,t} + (1 - \theta_j^{(0)}) \cdot r_{j,t-1} + \xi_{j,t}$$

where $\xi_{j,t} = \theta_j^{(0)} \cdot (u_{j,t} - u_{j,t-1})$

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
- The Bayesian framework can be generalized to a MA(H)
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
    - In this case, we have a MA(1) process
    - We still have unbiased estimates of β and α
    - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)

- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
  - The manager never learns $v_{j,t}$
  - But the manager observes noisy signals about $\eta_{j,t}$ and $\eta_{j,t-h}$
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
  - The manager never learns $v_{j,t}$
  - But the manager observes noisy signals about $\eta_{j,t}$ and $\eta_{j,t-h}$
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
  - The manager never learns $v_{j,t}$
  - But the manager observes noisy signals about $\eta_{j,t}$ and $\eta_{j,t-h}$
- The Bayesian framework can justify the 3-Step method as well

Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
  - The manager never learns $v_{j,t}$
  - But the manager observes noisy signals about $\eta_{j,t}$ and $\eta_{j,t-h}$
- The Bayesian framework can justify the 3-Step method as well
  - The manager observes separate signals for $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$

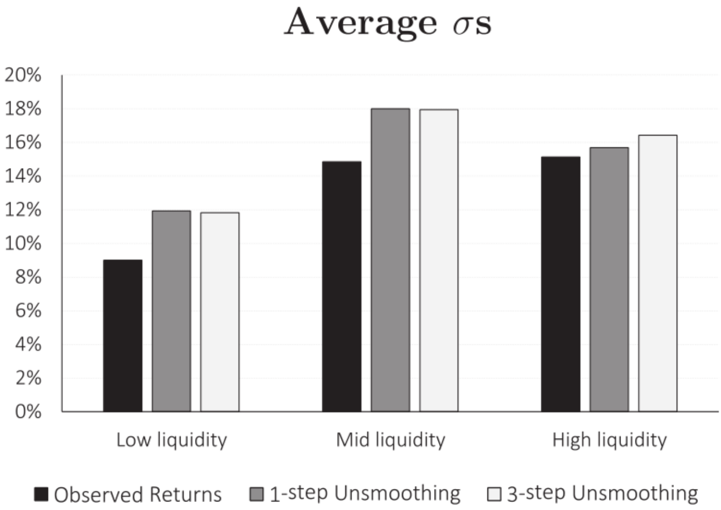
Smoothed Returns with Bayesian Fund Manager

- Our Bayesian smoothing process is a MA(1) except for $\xi_{j,t}$
- We can ignore the $\xi_{j,t}$ term (Couts, Gonçalves, Rossi (2020))
  - In this case, we have a MA(1) process
  - We still have unbiased estimates of β and α
  - The reason is that $\xi_{j,t}$ is uncorrelated with returns
- The Bayesian framework can be generalized to a MA(H)
  - At t, the manager learns $v_{j,t-H}$ (instead of $v_{j,t-1}$)
  - The manager can also observe noisy signals for $\eta_{j,t-h}$
- The Bayesian framework can also be generalized to a AR(L)
  - The manager never learns $v_{j,t}$
  - But the manager observes noisy signals about $\eta_{j,t}$ and $\eta_{j,t-h}$
- The Bayesian framework can justify the 3-Step method as well
  - The manager observes separate signals for $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$

Outline

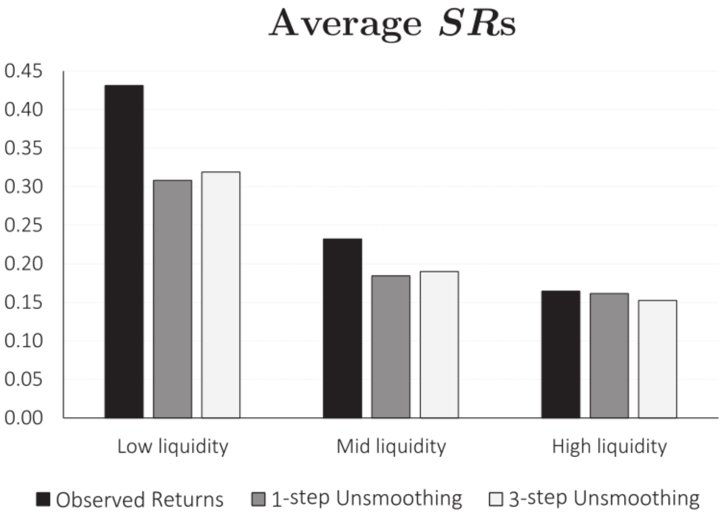
Hedge Funds

Hedge Funds



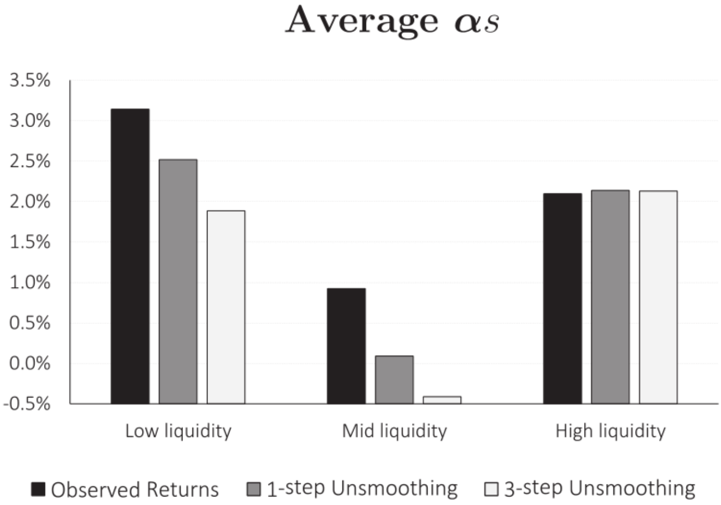
Source: Couts, Gonçalves, Rossi (2020)

Hedge Funds



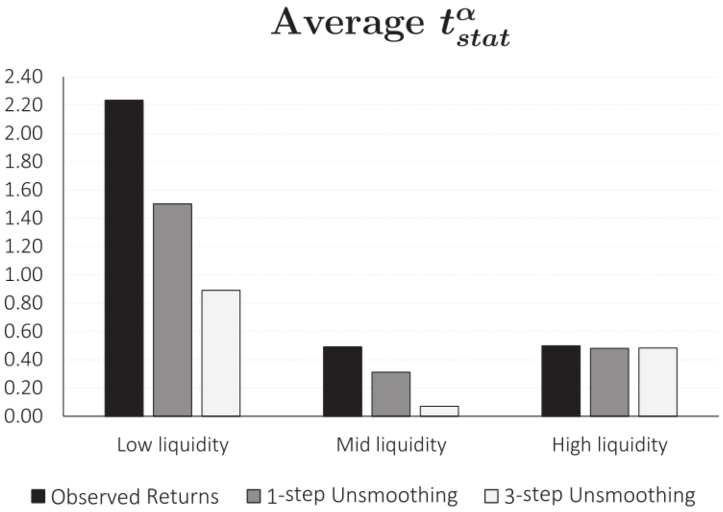
Source: Couts, Gonçalves, Rossi (2020)

Hedge Funds



Source: Couts, Gonçalves, Rossi (2020)

Hedge Funds



Source: Couts, Gonçalves, Rossi (2020)

Commercial Real Estate Funds

Commercial Real Estate Funds

Table 7
Risk and performance of private CRE funds

Statistics are related to	Raw performance		
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$
Observed, R_o	5.0%	13.1%	0.38
1-step, R_{1s}	5.0%	25.3%	0.20
3-step, R_{3s}	5.0%	24.1%	0.21

Source: Couts, Gonçalves, Rossi (2020)

Commercial Real Estate Funds

Table 7
Risk and performance of private CRE funds

Statistics are related to	Raw performance			1-factor model	
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34

Source: Couts, Gonçalves, Rossi (2020)

Commercial Real Estate Funds

Table 7
Risk and performance of private CRE funds

Statistics are related to	Raw performance			1-factor model		2-factor model		
	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	α	β_{re}	α	β_{re}	β_e
Observed, R_o	5.0%	13.1%	0.38	4.3%	0.07	4.0%	0.02	0.10
1-step, R_{1s}	5.0%	25.3%	0.20	2.7%	0.22	2.4%	0.15	0.15
3-step, R_{3s}	5.0%	24.1%	0.21	1.6%	0.34	0.8%	0.21	0.26

Source: Couts, Gonçalves, Rossi (2020)

Outline

Conclusion

- Illiquidity induces two major issues

- We explored (1) in this talk

- There are alternative approaches to deal with (1)

- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
  - Dimson regressions (Dimson (1979))
  - Nowcasted NAVs (Brown, Ghysels, Gredil (2023))
  - NPVs from cash flows
    (Gupta, Van Nieuwerburgh (2021), Korteweg, Nagel (2016, 2024))
- Need more work on how (2) affects performance measurement
  (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  - (1) Do not observe market values: hard to measure performance
  - (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
  - Dimson regressions (Dimson (1979))
  - Nowcasted NAVs (Brown, Ghysels, Gredil (2023))
  - NPVs from cash flows
    (Gupta, Van Nieuwerburgh (2021); Korteweg, Nagel (2016, 2024))
- Need more work on how (2) affects performance measurement
  (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
  - Dimson regressions (Dimson (1979))
  - Nowcasted NAVs (Brown, Ghysels, Gredil (2023))
  - NPVs from cash flows (Gupta, Van Nieuwerburgh (2021), Korteweg, Nagel (2016, 2024))
- Need more work on how (2) affects performance measurement (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
  - Dimson regressions (Dimson (1979))
  - Nowcasted NAVs (Brown, Ghysels, Gredil (2023))
  - NPVs from cash flows
    (Gupta, Van Nieuwerburgh (2021), Korteweg, Nagel (2016, 2024))
- Need more work on how (2) affects performance measurement
  (see Brown, Gonçalves, Hu (2024) for early work on that)

Conclusion

- Illiquidity induces two major issues
  (1) Do not observe market values: hard to measure performance
  (2) Cannot easily buy and sell assets: hard to manage allocations
- We explored (1) in this talk
  - Discussed smoothed returns and understated risk
  - Provided a solution: return unsmoothing methods
- There are alternative approaches to deal with (1)
  - Dimson regressions (Dimson (1979))
  - Nowcasted NAVs (Brown, Ghysels, Gredil (2023))
  - NPVs from cash flows
    (Gupta, Van Nieuwerburgh (2021), Korteweg, Nagel (2016, 2024))
- Need more work on how (2) affects performance measurement
  (see Brown, Gonçalves, Hu (2024) for early work on that)