A Theory of Managing the Liquidity Transformation Risks from Stale Pricing

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Abstract

This paper provides a simple two-period model to formalize and add structure to the economic intuition discussed in Couts (2025). It was originally included in an earlier version of that paper. As quoted in Couts (2025), “Open-end funds provide a liquidity transformation service by issuing and redeeming shares that are more liquid than their assets. However, because these assets are illiquid, managers need time to transfer capital to the underlying market. Liquidity buffers and liquidity restrictions enable this. Additionally, because of this illiquidity, their returns are predictable and susceptible to NAV-timing strategies that transfer wealth. I show that NAV-timing strategies appear profitable on paper and investors appear to follow these strategies. I also show liquidity restrictions provide a secondary benefit of protecting against these NAV-timing risks while liquidity buffers do not. In fact, liquidity buffers amplify them when added to liquidity restrictions.”

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1. Theoretical model

1.1. Setup

I create a two-period qualitative model to evaluate how stale pricing influences investor and fund behavior. The model motivates the empirical analysis and provides intuition into how discretionary liquidity restrictions and liquidity buffers jointly influence NAV-timing wealth transfers and incentives. The open-end fund is created in period 0 and assets are purchased at their true economic values as represented by Equation 1, where E denotes the economic value of the underlying assets. In period 1, the economic values of the assets purchased in period 0 are unobservable. After observing the reported returns from period 1, investors are able to submit subscription or redemption requests. The amount of capital either coming into or out of the fund in period 1 is the fund flow, FF_1, and is a percentage of the period 1 pre-fund flow total net asset value, $TNA_{1,a}$. The fund subsequently sells or purchases assets to meet the fund flow. The post fund flow total net asset value, $TNA_{1,b}$, incorporates the values of the assets sold or purchased. In period 2, the fund is liquidated and all assets are sold for their true economic values, TNA_2^E.

$$\begin{aligned} TNA_0^{Fund} &= TNA_0^E \\ &= \omega_0 \Pi_0^{MT} + \Pi_0^{BH} \end{aligned} \tag{1}$$

$$TNA_{1,a}^{Fund} = TNA_0^{Fund} R_1^{Fund} \tag{2}$$

$$TNA_{1,b}^{Fund} = TNA_0^{Fund} R_1^{Fund} (1 + FF_1) \tag{3}$$

$$TNA_2^{Fund} = TNA_2^E \tag{4}$$

There are two investors in the fund - a buy-and-hold investor, BH, and a market-timing investor, MT. As shown in Equation 1, the TNA value of the fund is equal to the combined investments of the market-timing and buy-and-hold investors in the fund during the period. Throughout the life of the fund, the market-timing investor has a percentage, ω_t, of his overall wealth, Π_t^{MT}, allocated to the fund while the rest of his wealth is invested in cash that provides a consistent risk-free return, R^f, of 1. The buy-and-hold investor maintains his entire wealth in the fund in each period until it is liquidated in period 2.

The model assumes the underlying assets have two characteristics associated with being illiquid. First, they have stale reported values. Second, large capital flows either into or out of the asset market have a temporary price impact. The level of staleness is represented by Θ, as shown in Equation 5, and influences the $TNA_{1,a}$ value at which investors enter and leave the fund. The assumption that capital flows have a price impact is reflected in Equation 6 where the transaction cost is a function of the period 1 fund flow. This equation implies that $\frac{\delta P}{\delta Q} > 0$ and $\frac{\delta^2 P}{\delta Q^2} > 0$, where P represents the price to purchase and sell assets in the underlying market. It is assumed the assets do not produce dividends. It is also assumed the assets will be sold for their true economic value in period 2. Lastly, the expected economic return for period 2 is 1. Because I am focusing on the effect of stale prices on investor and managerial behavior, investors act as if they are unaware of the impact their subscription or redemption requests will have on the future returns of the fund.

$$R_1^{Fund} = \left(R_1^E\right)^{(1-\Theta)}, \; where \, 0 < \Theta < 1 \tag{5}$$

$$Transaction\,Cost_1 = \psi \left(FF_1\right)^2, \; where \, 0 < \psi < 1 \tag{6}$$

1.2. Investor maximization

This analysis focuses on the way market-timing investors respond to stale NAVs. The market-timing investor chooses his portfolio allocations in period 1. His percent allocation to the open-end fund in period 1 is denoted by, ω_1. As reflected in Equation 7, the dollar amount of the fund flow equals the dollar change in the market-timing investor's allocation to the fund. $\omega_{1,a}$ represents the pre-fund flow allocation, and $\omega_{1,b}$ represents the post fund flow allocation. An adjustment cost is incurred by the market-timing investor for adjusting his allocations, as represented by Equation 8.

$$TNA_{1,a}TFF_1 = \Pi_1^{MT} \left(\omega_{1,b} - \omega_{1,a}\right) \tag{7}$$

$$Adjustment\,Cost_1 = \frac{\phi}{2}\left(\omega_{1,b} - \omega_{1,a}\right)^2 \tag{8}$$

The period 2 return for the portfolio held by the market-timing investor is the weighted average return of the open-end fund return and the risk-free rate less any adjustment costs. This is represented by Equation 9. The reported return of the fund in period 1 equals the reported TNA in period 1 divided by the TNA in period 0. Similarly, the reported return to the fund in period 2 equals the TNA of the fund in period 2 divided by the reported TNA of the fund in period 1, as shown in Equation 11.

$$R_2^{MT} = \left(\omega_{1,b} R_2^{Fund} + \left(1 - \omega_{1,b}\right) R^f\right) - \frac{\phi}{2}\left(\omega_{1,b} - \omega_{1,a}\right)^2 \tag{9}$$

$$R_1^{Fund} = \frac{TNA_1^{Fund}}{TNA_0^{Fund}} \tag{10}$$

$$R_2^{Fund} = \frac{TNA_2^{Fund}}{TNA_{1,b}^{Fund}} \tag{11}$$

The market-timing investor is interested in maximizing his period 2 return, and his choice variable is his allocation in the fund, $\omega_{1,b}$. As derived in Section A.2 of the Appendix, the optimal allocation for the market-timing investor is given by Equation 13. The economic interpretation of this equation is that the optimal allocation is chosen such that the marginal cost of adjusting the fund allocation from $\omega_{1,a}$ to $\omega_{1,b}$ equals the marginal benefit from the increased expected return associated with adjusting the fund allocation. By combining the optimal allocation with Equation 7, I obtain the optimal fund flow request, TFF_1, as shown in Equation 14. The proof of this derivation is provided in Section A.3 of the Appendix.

$$\max_{\{\omega_{1,b}\}} E_1\left(\left(\omega_{1,b} R_2^{Fund} + \left(1 - \omega_{1,b}\right) R^f\right) - \frac{\phi}{2}\left(\omega_{1,b} - \omega_{1,a}\right)^2\right) \tag{12}$$

$$\omega_{1,b} = \omega_{1,a} + \frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right) \tag{13}$$

$$TFF_1 = \frac{\Pi_1^{MT}}{TNA_{1,a}} \frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right) \tag{14}$$

The overall wealth transfer from existing investors to incoming investors is reflected in Equation 15. The wealth transfer experienced by the buy-and-hold investor depends on his period 0 percentage ownership of the fund, as shown in Equation 16. This wealth transfer amount assumes the market-timing investor is able to contribute or withdraw as much as he would like without restrictions. Three important outcomes of the model are demonstrated in Equation 16. First, wealth transfers increase with staleness in reported returns. Second, wealth transfers increase with the size of the economic return experienced in period 1. Third, wealth transfers increase with the magnitude of the fund flows allowed. The first two outcomes lead to **Predictions 1** and **2** listed below. The third outcome influences **Predictions 3** and **4** discussed below.

$$WT = TNA_0 \left(R_1^E - R_1^{Fund} \right) FF_1 \tag{15}$$

$$\begin{aligned} WT^{BH} &= \left(TNA_0 - \omega_0^{MT} \Pi_0^{MT} \right) \left(R_1^E - R_1^{Fund} \right) FF_1 \\ &= \left(TNA_0 - \omega_0^{MT} \Pi_0^{MT} \right) \left(R_1^E - \left(R_1^E \right)^{(1-\Theta)} \right) \frac{\Pi_1^{MT}}{TNA_1} \frac{1}{\phi} \left(\left(R_1^E \right)^{\Theta} - 1 \right) \end{aligned} \tag{16}$$

Prediction 1. *Investors will attempt to increase their holdings in funds after positive macroeconomic shocks.*

Prediction 2. *Investors will attempt to increase their holdings the most in funds with the highest past performance.*

1.3. Fund maximization (discretionary liquidity restrictions)

This analysis focuses on the way in which the fund responds to fund flow requests and how this influences wealth transfers. The model assumes the manager has full discretion over how much of the fund flow request to fulfill, as reflected in Equation 17. DFF_1 is the percentage of the total fund flow request, TFF_1, the manager chooses to fulfill. The maximization function of the manager has three components. The first component reflects the period 1 fee, which is a function of the amount of assets managed by the fund in that period. The second component reflects the impact the fund flow has on future returns. The loading on this component reflects the managers concern with the flow-performance relationship and their ultimate desire to form other funds in the future. The third component reflects the effect of not fulfilling investor subscription or redemption requests. It assumes investors are less willing to invest with managers tomorrow if their fund flow requests are not fulfilled in a timely manner today. Fund managers are interested in maximizing their lifetime earnings of fees.

After observing fund flow requests, the fund selects the optimal DFF_1 that will maximize its utility function. This maximization function is represented by Equation 18. As derived in Section A.2 of the Appendix, optimal percentage fund flow accepted by the fund is given in Equation 19. Accordingly, the fund's optimal fund flow occurs when the marginal cost of decreasing future fees, due to overpaying for assets, equals the marginal benefit from increasing contemporaneous and future fees, by fulfilling investor requests. It is important to note the impact on the contemporaneous fee reverses when the fund flow request is negative. Fulfilling a greater percentage of the requests has a marginal benefit during periods with a positive fund flow, while it has a marginal cost during periods with a negative fund flow.[1]

$$DFF_1 = \frac{FF_1}{TFF_1} \tag{17}$$

$$\max_{\{DFF_1\}} E_1 \left(\gamma_1 \left(1 + FF_1\right) - \frac{\gamma_2}{2} \psi \left(FF_1\right)^2 - \gamma_3 \left(TFF_1 - FF_1\right)^2 \right) \tag{18}$$

$$DFF_1 = \frac{\gamma_1 + \gamma_3 TFF_1}{\left(\gamma_2 \psi + \gamma_3\right) TFF_1} \tag{19}$$

The optimal fund flow in period 1 is jointly determined by the fund and the investors and is reflected in Equation 20. The wealth transfer from the buy-and-hold investor to the market-timing investor is obtained by substituting Equation 20 into Equation 16 as shown in Equation 21. Equation 21 depicts the importance of one of the novel characteristics of illiquidity - as illiquidity increases so does staleness, Θ, *and* price impact, ψ. Therefore, in this setup, the illiquidity has offsetting effects in the wealth transfer function. It creates both a wealth transfer risk and a wealth transfer mitigant. **Prediction 3** provides a hypothesis

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[1] The optimal discretionary fund flow, DFF_1, must be between 0 and 1. Alternatively, funds would be able to force investors to invest or divest at their discretion. As an example, if $\frac{\gamma_2}{2}\psi$ was sufficiently small and γ_1 was sufficiently large and positive, DFF_1 could mathematically be larger than 1. However, this does not occur in markets with illiquid assets. This is evidenced by the fact of both open-end and closed-end private equity funds regularly take multiple years to call and place all of their capital commitments.

consistent with this outcome.

$$FF_1 = \frac{\gamma_1 + \gamma_3 \frac{\Pi_1^{MT}}{TNA_{1,a}} \left(\frac{1}{\phi} \left(\left(R_1^E \right)^{\Theta} - 1 \right) \right)}{\left(\gamma_2 \psi + \gamma_3 \right)} \tag{20}$$

$$WT^{BH} = \left(TNA_0 - \omega_0^{MT} \Pi_0^{MT} \right) \left(R_1^E - \left(R_1^E \right)^{(1-\Theta)} \right) \cdot \frac{\gamma_1 + \gamma_3 \frac{\Pi_1^{MT}}{TNA_{1,a}} \left(\frac{1}{\phi} \left(\left(R_1^E \right)^{\Theta} - 1 \right) \right)}{\left(\gamma_2 \psi + \gamma_3 \right)} \tag{21}$$

Prediction 3. *Managers will limit capital flows the most at those times, and in those funds, where NAV-timing strategies appear most profitable.*

1.4. Fund maximization (liquidity buffers)

This analysis focuses on the effect of using liquidity buffers on wealth transfer outcomes. The choice variables for the market-timing investor and fund remain the same. The only difference is the fund uses a cash, liquidity buffer to mitigate the transaction cost associated with interacting with the underlying market, $\psi \left(FF_1 \right)^2$. As such, I remove the second component of fund's utility maximization equation, as shown in Equation 22. The new optimal DFF_1^{LB} is shown in Equation 23 and the proof is in Section A.6 of the Appendix. The new optimal fund flow is strictly larger than the one without a liquidity buffer. As such, the wealth transfer is strictly larger when the fund uses a liquidity buffer. I provide a proof and derivation of this in Section A.6. Liquidity buffers are effective at deterring the fragility risks associated with having to place or redeem capital too quickly in the underlying market, $\psi \left(FF_1 \right)^2$. However, they are ineffective at protecting against the fragility risks associated with stale pricing.

The purpose of this model is to evaluate the effect of using a liquidity buffer in combination with a discretionary liquidity restriction on the wealth transfer. The model does not attempt to find the optimal combination of liquidity buffers and liquidity restrictions. Accordingly, the liquidity buffer amount is not a choice variable in the model.

The result that wealth transfers are larger when liquidity buffers are used provides unique insights. First, funds create wealth transfers that would not otherwise exist when they use liquidity buffers and have stale NAVs. These wealth transfers increase strategic complementarities and first-mover advantages, which can destabilize these funds. In all, this evidence suggests the tools most commonly used to stabilize funds can be counterproductive and backfire in some situations. This evidence also supports **Prediction 4,** listed below.[2]

$$\max_{\left\{DFF_1^{LB}\right\}} E_1\left(\gamma_1\left(1+FF_1^{LB}\right)-\frac{\gamma_3}{2}\left(TFF_1-FF_1^{LB}\right)^2\right) \tag{22}$$

$$DFF_1^{LB}=\frac{\gamma_1+\gamma_3 TFF_1}{\gamma_3 TFF_1} \tag{23}$$

$$DFF_1^{LB} \gg DFF_1 \tag{24}$$

$$\left|FF_1^{LB}\right| \gg \left|FF_1\right| \tag{25}$$

$$WT^{BH,LB}=\left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)FF_1^{LB} \tag{26}$$

$$WT^{BH,LB}=\left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)\frac{\gamma_1+\gamma_3\frac{\Pi_1^{MT}}{TNA_{1,a}}\left(\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta}-1\right)\right)}{(\gamma_3)} \tag{27}$$

$$WT^{BH,LB} \gg WT^{BH} \tag{28}$$

Prediction 4. *Funds which use liquidity buffers instead of liquidity restrictions are more susceptible to runs and ultimate failure.*

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[2] The purpose of this model is to motivate the empirical analysis. However, it can be helpful to consider the implications for various corner solutions within the model. For instance, if the staleness factor (Θ) were 0, there would be no valuation staleness and no staleness enduced return predictability. Additionally, if Θ were 1, asset valuations would never be updated and they would simply reflect their acquisition values. If this were the case, investors would only realize the profits from stale NAVs when managers sell their assets. If the NAV-timing investor's adjustment cost (ϕ) were significantly large, it would be too costly for him to adjust his portfolio and he would not be able to take advantage of the predictability. Similarly, if ϕ were 0, he would reallocate all of his portfolio based on whether the NAV was undervalued or overvalued. Additionally, he would short the fund if it were overvalued and he would borrow additional capital to invest in the fund if the NAV was undervalued. Another way to think of the buy-and-hold invetor is a different NAV-timing investor with a relatively larger portfolio adjustment cost. This relatively larger adjustment cost could be due to any number of things including inattentiveness. Lastly, if the transaction cost (ψ) were 0, the fund would have no reason to put up its gates and the NAV-timing investor would be able to take full advantage of the stale pricing.

Appendix A Model Proof

This section provides the details of the model described in Section 1.

A.1 Wealth Transfer Proof

The overall wealth transfer, WT, from existing investors to incoming investors is represented by Equation A.2. As shown, the size of the wealth transfer is a function of two components: the staleness of the valuations, Θ, and the fund flow, FF_1. Wealth transfers are greater when either of these is greater. As shown in Equation A.2, this equation can be rewritten in terms of only the economic return and the fund flow by substituting the economic return for the fund return based on Equation A.1. Equations A.1 and A.2 correspond to Equations 5 and 15 in the main body of the paper.

$$R_1^{Fund} = \left(R_1^E\right)^{(1-\Theta)}, \; where \, 0 < \Theta < 1 \tag{A.1}$$

$$\begin{aligned} WT &= TNA_0 \left(R_1^E - R_1^{Fund}\right) FF_1 \\ &= TNA_0 \left(R_1^E - \left(R_1^E\right)^{(1-\Theta)}\right) FF_1 \end{aligned} \tag{A.2}$$

The wealth transfer of the buy-and-hold investor, WT^{BH}, can be determined by multiplying the overall wealth transfer by their percent ownership in the fund. This can be determined by calculating their percent ownership using Equation A.3 and adding it into Equation A.2 and combining like terms. Equations A.3 and A.4 correspond to Equations 1 and 16 in the main body of the paper.

$$TNA_0^{Fund} = \omega_0 \Pi_0^{MT} + \Pi_0^{BH} \tag{A.3}$$

$$\begin{aligned} WT^{BH} &= \frac{\left(TNA_0 - \omega_0^{MT}\Pi_0^{MT}\right)}{TNA_0} TNA_0 \left(R_1^E - \left(R_1^E\right)^{(1-\Theta)}\right) FF_1 \\ &= \left(TNA_0 - \omega_0^{MT}\Pi_0^{MT}\right) \left(R_1^E - \left(R_1^E\right)^{(1-\Theta)}\right) FF_1 \end{aligned} \tag{A.4}$$

A.2 Investor's Response to Stale Pricing Incentives

In this model, the market-timing investor maximizes the period 2 expected return to his overall investment portfolio, R_2^{MT}, by selecting his optimal period 1 fund allocation, $\omega_{1,b}$. This maximization function is subject to a an adjustment cost, $\frac{\phi}{2}\left(\omega_{1,b}-\omega_{1,a}\right)^2$, as reflected in Equation A.5. The expectation notation is dropped in the second line of the derivation to reduce clutter. The optimal fund allocation be obtained by taking the first order condition and solving for the post fund flow allocation. The expected period 2 fund return as a function of the economic returns is then substituted into this equation to obtain the optimal allocation as reflected by Equation A.9.[3] As noted in the main body of the paper, $E_1\left(R_2^E\right) = 1$. Equation A.9 corresponds to Equation 13 in the main body of the paper.

$$\begin{aligned}\max_{\{\omega_{1,b}\}} E_1\left(R_2^{MT}\right) &= \max_{\{\omega_{1,b}\}} E_1\left(\left(1-\omega_{1,b}\right)R_2^{rf} + \omega_{1,b}R_2^{Fund} - \frac{\phi}{2}\left(\omega_{1,b}-\omega_{1,a}\right)^2\right)\\ &= \max_{\{\omega_{1,b}\}} R_2^{rf} - \omega_{1,b}R_2^{rf} + \omega_{1,b}R_2^{Fund} - \frac{\phi}{2}\left(\omega_{1,b}-\omega_{1,a}\right)^2\end{aligned} \tag{A.5}$$

First Order Condition:

$$0 = R_2^{Fund} - R_2^{rf} - \phi\left(\omega_{1,b}-\omega_{1,a}\right) \tag{A.6}$$

$$\phi\left(\omega_{1,b}-\omega_{1,a}\right) = R_2^{Fund} - 1 \tag{A.7}$$

$$\omega_{1,b}-\omega_{1,a} = \frac{1}{\phi}\left(R_2^{Fund} - 1\right) \tag{A.8}$$

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[3] As noted in the main body of the paper, investors act as if they are unaware of the impact their subscription or redemption requests will have on the future returns of the fund. Their subscription or redemption requests will impact the fund return in two ways. First, it will lower the future returns of the fund through the transaction costs the fund will incur due to the illiquidity of the assets. Second, it will lower the returns of the fund by increasing or decreasing the dollar basis of the fund. Neither of these change the conclusions of the model.

$$\begin{aligned}\omega_{1,b} &= \omega_{1,a} + \frac{1}{\phi}\left(\frac{R_1^E R_2^E}{R_1^{Fund}} - 1\right) \\ &= \omega_{1,a} + \frac{1}{\phi}\left(\frac{R_1^E R_2^E}{\left(R_1^E\right)^{(1-\Theta)}} - 1\right) \\ &= \omega_{1,a} + \frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right)\end{aligned} \tag{A.9}$$

A.3 Fund Flow Requested Proof

Based on the market-timing investor's optimal fund allocation, the requested fund flow, TFF_1, can be derived as follows. The market-timing investor's post fund flow fund allocation is substituted into the equation which equates the dollar change in the market-timing investors allocation to the fund with the dollar change in the total net assets of the fund as shown in Equation A.10. Both sides are then divided by the pre-fund flow Total Net Assets leaving the optimal Fund Flow percentage as shown in Equation A.11 below. Equations A.10 and A.11 correspond to Equations 7 and 14 in the main body of the paper.

$$\begin{aligned}TNA_{1,a}TFF_1 &= \Pi_1^{MT}\left(\omega_{1,b} - \omega_{1,a}\right) \\ &= \Pi_1^{MT}\left(\omega_{1,a} + \frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right) - \omega_{1,a}\right) \\ &= \Pi_1^{MT}\left(\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right)\right)\end{aligned} \tag{A.10}$$

$$TFF_1 = \frac{\Pi_1^{MT}}{TNA_{1,a}}\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right) \tag{A.11}$$

Based on this fund flow, the overall wealth transfer and the wealth transfer experienced by the buy-and-hold investor would be as follows. Equation A.13 corresponds with Equation 16 in the main body of the paper.

$$E_1\left(WT\right) = E_1\left(TNA_0\left(R_1^E - R_1^{Fund}\right)\frac{\Pi_1^{MT}}{TNA_{1,a}}\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right)\right) \tag{A.12}$$

$$E_1\left(WT^{BH}\right) = E_1\left(\left(TNA_0 - \omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E - \left(R_1^E\right)^{(1-\Theta)}\right)\frac{\Pi_1^{MT}}{TNA_{1,a}}\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta} - 1\right)\right) \tag{A.13}$$

A.4 Fund's Response to Price Impact Incentives

In this model, funds maximize their lifetime earning of fees, π, by selecting their optimal period 1 discretionary fund flow, DFF_1. This discretionary fund flow is a percentage of the fund flow requested by investors as reflected by Equation A.14. This maximization function is subject to a a transaction cost, $\frac{\gamma_2}{2}\psi\left(FF_1\right)^2$, and an investor sentiment cost, $\frac{\gamma_3}{2}\left(TFF_1 - FF_1\right)^2$, representing an investors desire to have their fund flow requests fulfilled in a timely manner. These constraints are reflected in the fund's maximization function shown in Equation A.5. The expectation notation is dropped in the second line of the derivation to reduce clutter.

This equation can be expanded by substituting the product of the discretionary fund flow and the total fund flow requested for the actual fund flow based on Equation A.14. The optimal discretionary fund flow can then be obtained by taking the first order condition and solving for this fund flow ratio. As noted in the main body of the paper, DFF_1 must be between 0 and 1. Otherwise, the fund could force investors to invest beyond their desired allocations. Equations A.14, A.15, and A.19 correspond to Equations 17, 18, and 19 in the main body of the paper.

$$DFF_1 = \frac{FF_1}{TFF_1} \tag{A.14}$$

$$\begin{aligned}\max_{\{DFF_1\}} E_1\left(\pi\right) &= \max_{\{DFF_1\}} E_1\left(\gamma_1 FF_1 - \frac{\gamma_2}{2}\psi\left(FF_1\right)^2 - \frac{\gamma_3}{2}\left(TFF_1 - FF_1\right)^2\right)\\ &= \max_{\{DFF_1\}} \gamma_1 DFF_1 TFF_1 - \frac{\gamma_2}{2}\psi\left(DFF_1 TFF_1\right)^2 - \frac{\gamma_3}{2}\left(TFF_1 - DFF_1 TFF_1\right)^2\end{aligned} \tag{A.15}$$

First Order Condition:

$$\begin{aligned}0 &= \gamma_1 TFF_1 - \gamma_2\psi DFF_1 TFF_1 TFF_1 + \gamma_3\left(TFF_1 - DFF_1 TFF_1\right)TFF_1\\ &= \gamma_1 TFF_1 - \gamma_2\psi DFF_1\left(TFF_1\right)^2 + \gamma_3\left(TFF_1\right)^2 - \gamma_3 DFF_1\left(TFF_1\right)^2\end{aligned} \tag{A.16}$$

$$\gamma_2\psi DFF_1\left(TFF_1\right)^2 + \gamma_3 DFF_1\left(TFF_1\right)^2 = \gamma_1 TFF_1 + \gamma_3\left(TFF_1\right)^2 \tag{A.17}$$

$$DFF_1\left(\gamma_2\psi+\gamma_3\right)\left(TFF_1\right)^2=\gamma_1 TFF_1+\gamma_3\left(TFF_1\right)^2 \tag{A.18}$$

$$DFF_1=\frac{\gamma_1+\gamma_3 TFF_1}{\left(\gamma_2\psi+\gamma_3\right)TFF_1} \tag{A.19}$$

A.5 Joint Solutions: Investor and Fund Responses

Based on Equation A.14, the chosen fund flow can be derived by multiplying the fund's optimal discretionary fund flow, given by Equation A.19, with the investor's optimal fund allocation, given by Equation A.11. Equation A.20 corresponds to Equation 20 in the main body of the paper.

$$FF_1=\frac{\gamma_1+\gamma_3\frac{\Pi_1^{MT}}{TNA_{1,a}}\left(\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta}-1\right)\right)}{\left(\gamma_2\psi+\gamma_3\right)} \tag{A.20}$$

The wealth transfer can then be rewritten in terms of the investor and fund optimizations by substituting the variables for the fund flow based on Equation A.20. The wealth transfer with liquidity restrictions is strictly less than the one without discretionary liquidity restrictions. This is based on the assumptions that both stale pricing, Θ, and transaction costs, ψ, increase as illiquidity increases. Equation A.21 corresponds to Equation 21 in the main body of the paper.

$$WT^{BH}= \left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)\frac{\gamma_1+\gamma_3\frac{\Pi_1^{MT}}{TNA_{1,a}}\left(\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta}-1\right)\right)}{\left(\gamma_2\psi+\gamma_3\right)} \tag{A.21}$$

A.6 Fund's Response Using Liquidity Buffers

The fund's optimal discretionary fund flow assuming it uses a liquidity buffer can be derived as follows. The component related to the transaction cost, ψ , is removed from the optimization function given the fund uses a liquidity buffer to not incur these costs. The wealth transfer with liquidity buffers added to liquidity restrictions is strictly greater than the one with only liquidity restrictions. Equation A.29 corresponds to Equation 27 in the main body of the paper.

- **Fund Optimization DFF selection:**

$$\begin{aligned}\max_{\{DFF_1\}} E_1(\pi) &= \max_{\{DFF_1^{LB}\}} E_1\left(\gamma_1\left(1+FF_1^{LB}\right)-\frac{\gamma_3}{2}\left(TFF_1-FF_1^{LB}\right)^2\right)\\ &= \max_{\{DFF_1^{LB}\}} \gamma_1\left(1+TFF_1DFF_1^{LB}\right)-\frac{\gamma_3}{2}\left(TFF_1-TFF_1DFF_1^{LB}\right)^2\end{aligned} \tag{A.22}$$

First Order Condition:

$$\begin{aligned}0 &= \gamma_1 TFF_1+\gamma_3\left(TFF_1-DFF_1^{LB}TFF_1\right)TFF_1\\ &= \gamma_1 TFF_1+\gamma_3\left(TFF_1\right)^2-\gamma_3 DFF_1^{LB}\left(TFF_1\right)^2\end{aligned} \tag{A.23}$$

$$DFF_1^{LB}\gamma_3\left(TFF_1\right)^2=\gamma_1 TFF_1+\gamma_3\left(TFF_1\right)^2 \tag{A.24}$$

$$DFF_1^{LB}=\frac{\gamma_1+\gamma_3 TFF_1}{\gamma_3 TFF_1} \tag{A.25}$$

$$DFF_1^{LB}=\frac{\gamma_1+\gamma_3 TFF_1}{\gamma_3 TFF_1}\gg\frac{\gamma_1+\gamma_3 TFF_1}{\left(\gamma_2\psi+\gamma_3\right)TFF_1}=DFF_1 \tag{A.26}$$

$$DFF_1^{LB}\gg DFF_1 \tag{A.27}$$

$$\left|FF_1^{LB}\right|\gg\left|FF_1\right| \tag{A.28}$$

- **Wealth Transfer**

$$WT^{BH,LB}=\left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)\cdot\frac{\gamma_1+\gamma_3\frac{\Pi_1^{MT}}{TNA_{1,a}}\left(\frac{1}{\phi}\left(\left(R_1^E\right)^{\Theta}-1\right)\right)}{\left(\gamma_3\right)} \tag{A.29}$$

$$\begin{aligned}WT^{BH,LB}=\left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)FF_1^{LB}\gg\\ \left(TNA_0-\omega_0^{MT}\Pi_0^{MT}\right)\left(R_1^E-\left(R_1^E\right)^{(1-\Theta)}\right)FF_1=WT^{BH}\end{aligned} \tag{A.30}$$

$$WT^{BH,LB}\gg WT^{BH} \tag{A.31}$$

References

Couts, S. J., 2025. Liquidity transformation risks and stabilization tools: Evidence from open-end private equity real estate funds. Real Estate Economics (forthcoming) .